Bayesian parameter estimation for the SWIFT model of eye-movement control during reading

Stefan A. Seelig (stefan.seelig@uni-potsdam.de)
Department of Psychology, University of Potsdam

Maximilian M. Rabe (maximilian.rabe@uni-potsdam.de)
Department of Psychology, University of Potsdam

Noa Malem-Shinitski
Institute of Mathematics, University of Potsdam

Sebastian Reich
Institute of Mathematics, University of Potsdam
Karl-Liebknecht-Str. 24-25, 14469 Potsdam, Germany

Ralf Engbert
Department of Psychology, University of Potsdam
Karl-Liebknecht-Str. 24-25, 14469 Potsdam, Germany

Abstract

Dynamical models are increasingly contributing to the development of cognitive theory. Here we discuss an example for eye-movement control during reading. The SWIFT model (Engbert et al., 2005) is a stochastic dynamical system that predicts spatial fixation positions in a given text as well as fixation durations. We exploit the sequential nature of the likelihood for dynamical models. The likelihood function is a combination of spatial and temporal likelihood. While the spatial part is a pseudo-marginal likelihood, the temporal likelihood is obtained by numerical approximation. We use a fully Bayesian framework for parameter inference using an adaptive Markov Chain Monte Carlo (MCMC) procedure. As a result, we obtain model parameter estimates and credibility intervals on the level of individual readers. Interindividual parameter variations capture key features of the behavioral variability of eye movements observed in reading experiments.

Keywords: eye movements; reading; Bayesian inference; Markov Chain Monte Carlo; individual differences

Background

Reading is characterized by the successful coordination between key cognitive and motor subsystems, e.g., visual information processing, attention, word recognition, and saccade programming. Even during reading of simple texts, there is considerable stochastic variability in fixation positions and fixation durations (Fig. 1). One motivation for the development of mathematical models of eye-movement control during reading is to explain the observed variability.

The SWIFT Model

The SWIFT (saccade generation with inhibition by foveal targets, Engbert, Nuthmann, Richter, & Kliegl, 2005) is a spatially-extended dynamical system that seeks to explain

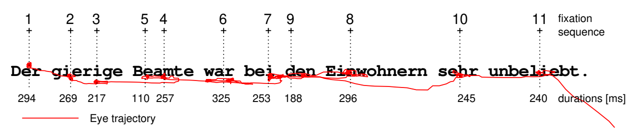

Figure 1: Sequence of fixations during reading. The eye trajectory (red line) is segmented into alternating periods of stationarity (fixations; dotted lines, number indicates order, durations beneath) and quick repositioning (saccades). The sequence contains refixations (3,5), word skipping (8) and regression (9) to a previous word.

saccadic selection by the temporal evolution of an activation field. The lexical processing of each word i in a given sentence is represented by an activation variable $a_i(t)$. The target selection probability $\pi_n(t)$ for word n at time t is computed from relative activation. As time evolves, relative activations change to produce a continuous-time process that predicts saccadic selection over time, i.e.,

$$\pi_n(t) = \frac{[a_n(t)]^\gamma}{\sum_j [a_j(t)]^\gamma} , \tag{1}$$

where γ is a weighting exponent. Fixation durations can be approximated (at first order) by an uncorrelated random process. To introduce word difficulty effects, however, we modulate fixation duration by a process called foveal inhibition that delays upcoming saccades to prolong ongoing fixations. A simulated trajectory of the model is shown in Figure 2.

Parameter Estimation

The Likelihood Function

For parameter estimation, the likelihood of fixation locations (spatial contribution) and fixation durations (temporal contribution) must be calculated incrementally with respect to all previous events in the fixation sequence. We recently showed that

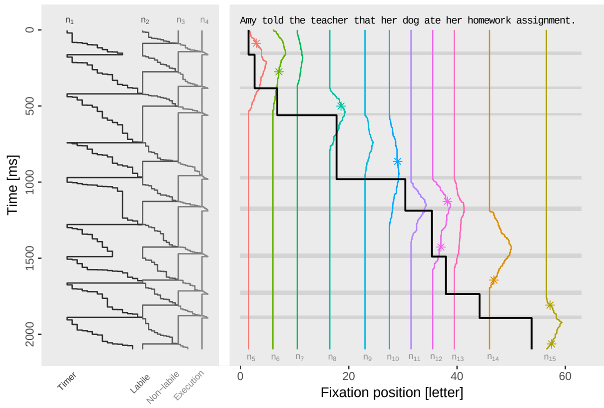

Figure 2: Simulation example for the SWIFT model. Gaze position (black line, right) is shifted across the sentence, driven by relative word activations (colored lines, right) at intervals determined by cascading random saccade timers (grey lines, left).

a combination of methods of pseudo-marginal likelihoods with approximate Bayesian computation (Toni, Welch, Strelkowa, Ipsen, & Stumpf, 2008) is a viable approach to likelihood computation for the SWIFT model (Seelig et al., 2019).

The contributions of the temporal and spatial parts of the likelihood function are shown in Figure 3, where one parameter was varied and the likelihood for simulated data with known parameters was evaluated.

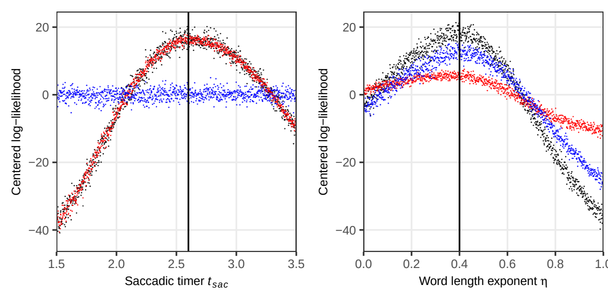

Figure 3: **Temporal**, **spatial**, and **combined** likelihood profiles for a simulated dataset (true parameters indicated by vertical lines). While the saccade timer (left) only influences the temporal likelihood, the word length exponent (right) affects both components.

Results

We implemented a fully Bayesian framework for parameter inference (Schütt et al., 2017) and used an adaptive MCMC procedure, the DREAM framework with improvements (ter Braak & Vrugt, 2008). For parameter estimation, we used eye tracking data of 36 participants who read 150 single sentences each. For every participant 70% of the data were used during the estimation. The remaining 30% were then compared with simulated data sets which were based on point estimates of the obtained posterior parameter distributions. We compared typical measures of fixation durations (contingent on saccade programming) and fixation probabilities (relating to oculomotor behavior and target selection). The comparisons indicate a remarkable agreement of artificial and experimental data.

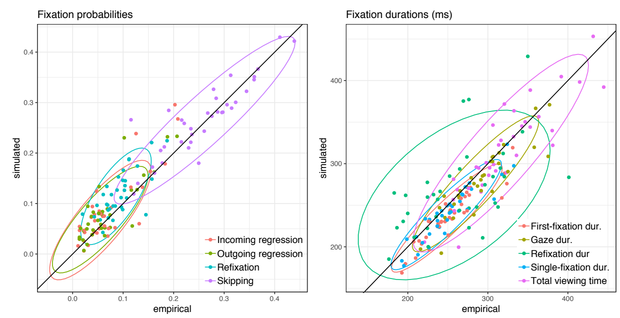

Figure 4: Relationship between fixation probabilities (left) and mean fixation durations (right) of simulated and experimental data. Each datapoint represents one participant.

Conclusion

We studied Bayesian parameter inference for a dynamical cognitive model of eye-movement control during reading. Using an adapative MCMC framework, we were able to estimate model parameters on the level of individual readers. Simulation on a test data set indicate that a high correlation between important measures for experimental and simulated data was obtained.

Acknowledgments

This work is part of CRC1294 Data Assimilation and of CRC1287 Variability in Language, funded by Deutsche Forschungsgemeinschaft.

References

Engbert, R., Nuthmann, A., Richter, E. M., & Kliegl, R. (2005). Swift: a dynamical model of saccade generation during reading. *Psychological Review*, *112*(4), 777–813.

Schütt, H. H., Rothkegel, L. O., Trukenbrod, H. A., Reich, S., Wichmann, F. A., & Engbert, R. (2017). Likelihood-based parameter estimation and comparison of dynamical cognitive models. *Psychological Review*, *124*(4), 505–524.

Seelig, S. A., Rabe, M. M., Malem-Shinitski, N., Risse, S., Reich, S., & Engbert, R. (2019). Bayesian parameter estimation for the swift model of eye-movement control during reading. *arXiv preprint arXiv:1901.11110*.

ter Braak, C. J., & Vrugt, J. A. (2008). Differential evolution markov chain with snooker updater and fewer chains. *Statistics and Computing*, *18*(4), 435–446.

Toni, T., Welch, D., Strelkowa, N., Ipsen, A., & Stumpf, M. P. (2008). Approximate bayesian computation scheme for parameter inference and model selection in dynamical systems. *Journal of the Royal Society Interface*, *6*(31), 187–202.